Chapter 7

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ENERGY OF A SYSTEM

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Lecture 04

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Work And Elastic Potential Energy

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Work and Elastic Potential Energy

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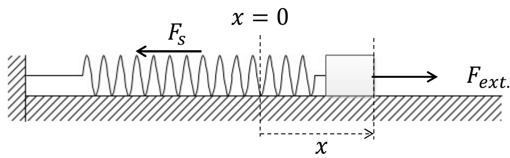
Spring force (Hooke's law)

The force exerted by the spring is given by hook's law:

$$F_s = -\kappa x$$



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Work and Elastic Potential Energy

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- x is the position of the block with respect to the equilibrium position ($x = 0$).
- κ is called the spring constant or force constant and measures the stiffness of the spring (in N/m).
- Negative sign: because the force exerted by the spring is always directed opposite to the displacement from equilibrium.
- The spring force is sometimes called the restoring force.

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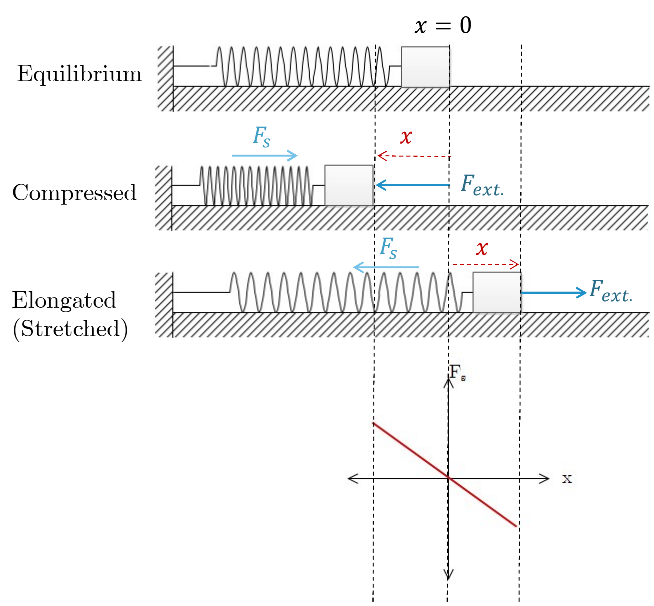

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Work and Elastic Potential Energy

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The force varies with position.

Calculate the work as the block moves from x_i to x_f.

$$W_s = \int_{x_i}^{x_f} -\kappa x \, dx$$

$$W_s = -\frac{1}{2}\kappa\left(x_f^2 - x_i^2\right)$$

$$\boxed{W_s = -\left(\frac{1}{2}\kappa x_f^2 - \frac{1}{2}\kappa x_i^2\right)}$$

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Wok and Elastic Potential Energy

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The applied force is equal and opposite to the spring force.

For any displacement, the work done by the applied force is

$$W_{ext.} = \frac{1}{2}\kappa\left(x_f^2 - x_i^2\right)$$

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Work and Elastic Potential Energy

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Elastic Potential Energy is associated with a spring.

The work done by the spring force is:

$$W_s = -\left(\frac{1}{2}\kappa x_f^2 - \frac{1}{2}\kappa x_i^2\right)$$

The expression ($\frac{1}{2}\kappa x^2$) is the elastic potential energy:

$$U_s = \frac{1}{2}\kappa x^2$$

The elastic potential energy can be thought of as the energy stored in the deformed spring.

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Work and Elastic Potential Energy

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The elastic potential energy stored in a spring:

- Is zero whenever the spring is not deformed (stretched or compressed)(U = 0 when x = 0).
- Is a maximum when the spring has reached its maximum extension or compression.
- Is a scalar quantity.
- Is always positive.
- Has units: $\frac{N}{m}.m^2 = N.m = Joule$

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Work and Elastic Potential Energy

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The work done by the spring force is:

$$W_s = -\frac{1}{2}\kappa\left(x_f^2 - x_i^2\right)$$

The work done by the spring force along a closed path is zero $\left(x_f = x_i\right)$.

The work done by the spring force is path independent.

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Work and Elastic Potential Energy

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The work done by the spring force is:

$$W_s = -\left(\frac{1}{2}\kappa x_f^2 - \frac{1}{2}\kappa x_i^2\right)$$

$$W_s = -(U_{sf} - U_{si})$$

$$W_s = -\Delta U$$

The work done by the spring force equals the decrease in potential energy.

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Measuring κ for a Spring

Saturday, 30 January, 2021 15:12

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.

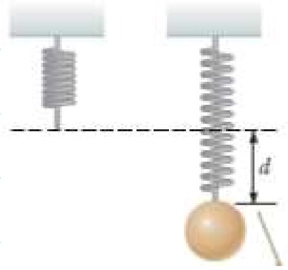
R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.

J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY,2014.

H. D. Young and R. A. Freedman, *University Physics with Modern Physics*, 14th ed., PEARSON, 2016.

H. A. Radi and J. O. Rasmussen, *Principles of Physics For Scientists and Engineers*, 1st ed., SPRINGER, 2013.

- If a spring is stretched $2\ cm$ by a suspended object having a mass of $0.55\ kg$, what is the force constant of the spring?
- How much work is done by the spring on the object as it stretches through this distance?



SOLUTION

Because the object is in equilibrium, the net force on it is zero and the upward spring force balances the downward gravitational force $m\vec{g}$.

Apply the particle in equilibrium model to the object:

$$\vec{F}_s + m\vec{g} = 0 \Rightarrow F_s - mg = 0 \Rightarrow F_s = mg$$

Apply Hooke's law to give $F_s = kd$and solve for k :

$$k = \frac{mg}{d} = \frac{(0.55kg)\ (9.80m/s^2)}{2.0 \times 10^{-2}m} = 2.7 \times 10^2 N/m$$

(B) How much work is done by the spring on the object as it stretches through this distance?

SOLUTION

The work done by the spring on the object:

$$W_s = 0 - \frac{1}{2}kd^2 = -\frac{1}{2}(2.7 \times 10^2 N/m)(2.0 \times 10^{-2}m^2) = -5.4 \times 10^{-2} J$$

As the object moves through the (2 cm) distance, the gravitational force also does work on it. This work is positive because the gravitational force is downward and so is the displacement of the point of application of this force.

Evaluate the work done by the gravitational force on the object:

$$W = \vec{F} \cdot \Delta\vec{r} = mgd \cos 0 = mgd$$

$$= (0.55kg)(9.80m/s^2)(2.0 \times 10^{-2}m) = 1.1 \times 10^{-1} J$$

Work done by a spring

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The force an ideal spring exerts on an object is given by: $F_s = -\kappa x$, where x measures the displacement of the object from its equilibrium position. If $\kappa = 60\ N/m$, how much work is done by this force as the object moves from $x = -0.2\ m$ to $x = 0\ m$?

$$W_s = \int_{-0.2}^{0} -60x\, dx = -30x^2\Big|_{-0.2}^{0} = -30 \times (0^2 - (-0.2)^2) = 1.2J$$